PART II: FOUNDATIONS OF MONETARY ECONOMICS IN AN OPEN ECONOMY

CHAPTER 5: THE BASIC MONETARY MODEL

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- **Up to now, Chapters 1 to 4, are real models abstracted from nominal (monetary) considerations. Chapters 5 to 9 looks at the foundations of monetary economics in an open economy.**

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6.1 El enfoque Monetario de la balanza de pagos.

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1. INTRODUCTION

- Chapter 5 introduces money into the endowment economy of Chapter 1.
- Money is introduced in such a way that it acts as a "veil" in the sense that the economy's real variables (i.e., consumption and the external accounts) are independent of the path of monetary variables such as the money supply and the real exchange rate.
- This will allow us to understand monetary considerations in isolation from the rest of the economy.
- Section 2 defines predetermined and flexible exchange rate regimes.
- Section 3 shows that monetary and exchange rate policy are both *neutral* (changes in the **level** of the money supply or the exchange rate do not affect the real sector) and *superneutral* (changes in the **growth rate** of the money supply or the exchange rate do not affect the real sector).
- Section 4 shows the fundamental equivalence between predetermined and flexible exchange rates.
- Section 5 analyzes the different response of the economy to money shocks depending on the exchange rate regime.
- Section 6 introduces a cash-in-advance constraint to give a different (from money in the utility function) motive to hold money balances.
- In sum, this chapter focuses on the simplest monetary model of a small open economy in which money is a veil in the sense that monetary/exchange rate policy does not affect the real sector. Subsequent chapters will introduce various frictions into this benchmark model that will "remove the veil" and allow monetary variables to affect the real sector.

2. THE BASIC MONETARY MODEL

- Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers who are endowed with perfect foresight.
- The economy is perfectly integrated with the rest of the world in both goods and capital markets.
- There is only one (tradable and non-storable) good, whose price is given by the Law of One Price $(P_t = E_t P_t^*)$.
- The economy receives a flow endowment of the good (y_t).
- The international real interest rate (r) is given and constant over time.

2.1 CONSUMER'S PROBLEM
2.1.1 BUDGET CONSTRAINTS

① $A_t = M_t + E_t B_t^*$ A_t = Financial Assets

M_t = domestic Money

B_t^* = Int'l Bond in Foreign Currency

E_t = Nominal Exchange Rate (Local/Foreign)

② Law of One Price

$P_t = E P_t^*$ P_t = Domestic price of the good; P_t^* = Foreign price of the good

③ $\pi_t = \varepsilon_t + \pi_t^*$; $\pi_t = \frac{\dot P_t}{P_t}$ $\varepsilon_t = \frac{\dot E_t}{E_t}$

$\pi_t^* = \frac{\dot P_t^*}{P_t}$

De ② $\ln P_t = \ln E_t + \ln P_t^*$

Derivando

$$\frac{\frac{dP_t}{dt}}{P_t} = \frac{\frac{dE_t}{dt}}{E_t} + \frac{\frac{dP_t^*}{dt}}{P_t^*}$$

$$\pi_t = \varepsilon_t + \pi_t^*$$

π_t = tasa de inflación doméstica ; ε_t = tasa de cambio del tipo de cambio

π_t^* = tasa de inflación extranjera

④ $a_t = m_t + b_t$ $a_t = \frac{A_t}{P_t}$; $m_t = \frac{M_t}{P_t}$; $b_t = \frac{B_t^*}{P_t^*}$ (en términos reales)

Restricción Presupuestal

⑤ $\frac{dA_t}{dt} = \dot A_t = E_t i_t^* B_t^* + \dot E_t B_t^* + P_t y_t + P_t z_t - P_t c_t$

(Intereses; Cap Gains; Ingreso Prod del Bien; Transf Gov; Consumo)

i_t^* = Nominal rate (foreign)

$\frac{d}{dt}\left[E_t B_t^*\right] = E_t \cdot \frac{dB_t^*}{dt} + B_t^* \frac{dE}{dt}$; $\frac{dB_t^*}{dt} = B_t^* i_t^*$

$= E_t B_t^* i_t^* + \dot E_t B_t^*$

$\dot a_t \neq \frac{\dot A_t}{P_t}$

⑥ $\frac{\dot A_t}{P_t} = (i_t^* + \varepsilon_t) b_t + y_t + z_t - c_t$

⑤' $b_t = \frac{B_t^*}{P_t^*}$; $\frac{B_t^*(E_t i_t^* + \dot E_t)}{P_t} = ?$

$\varepsilon_t = \frac{\dot E_t}{E_t}$;

$\frac{B_t^*}{P_t} = \frac{b_t P_t^*}{P_t} = \frac{b_t}{E_t}$ de ⑤'

2.1.1 BUDGET CONSTRAINTS (CONT.)

$$\frac{B_t^*}{P_t}\left[E_t i_t^* + \dot E_t\right] = \frac{b}{E_t}\left[E_t i_t^* + E_t \varepsilon_t\right]$$

$$= b(i_t^* + \varepsilon_t) \quad (5')$$

$$\therefore \quad \frac{\dot A_t}{P_t} = (i_t^* + \varepsilon_t) b_t + y_t + z_t - c_t \quad (6)$$

Ya que $P_t = E_t P_t^*$ $\Rightarrow$ $a_t = \frac{A_t}{P_t} = \frac{A_t}{E_t P_t^*}$ (6')

De (6') se tiene:

$$\ln a_t = \ln A_t - (\ln E_t + \ln P_t^*)$$

$$\frac{\dot a_t}{a_t} = \frac{\dot A_t}{A_t} - \frac{\dot E_t}{E_t} - \frac{\dot P_t^*}{P_t^*} = \frac{\dot A_t}{A_t} - \varepsilon_t - \pi_t^*$$

$$a_t = \frac{A_t}{P_t} \qquad A_t = P_t a_t$$

$$\dot a = a \frac{\dot A_t}{P_t a} - a(\varepsilon_t + \pi_t^*) = \frac{\dot A_t}{P_t} - (\varepsilon_t + \pi_t^*) a_t \quad (7)$$

$$\Rightarrow \frac{\dot A_t}{P_t} = \dot a_t + (\varepsilon_t + \pi_t^*) a_t \quad (7')$$

Sustituyendo (7') en (6)

$$\dot a_t + (\varepsilon_t + \pi_t^*) a_t = (i_t^* + \varepsilon_t) b_t + y_t + z_t - c_t$$

Recordar que $a_t = m_t + b_t$ $\Rightarrow$ $b_t = a_t - m_t$

$$\dot a_t = -(\varepsilon_t + \pi_t^*) a + (i^* + \varepsilon)(a - m) + y + z - c$$

$i_t = i^* + \varepsilon$ — la Tasa doméstica es igual a la tasa extranjera más el cambio en el tipo de cambio (depreciación)

y también

$i^* = r + \pi^*$ — la tasa nominal extranjera es igual a la tasa real r extranjera + la inflación extranjera

$$i = i^* + \varepsilon = r + \pi^* + \varepsilon$$

$$\dot a = a(i^* + \varepsilon - \varepsilon - \pi^*) - (i^* + \varepsilon) m + y + z - c$$

$$\dot a = (i^* - \pi^*) a + y + z - c - (i^* + \varepsilon) m \quad (8)$$

Ya que $i^* - \pi^* = r$ y $i^* + \varepsilon = i$

$$\Rightarrow \quad \dot a_t = r a_t + y_t + z_t - c_t - i m_t \quad (9)$$

El cambio en el # de activos es igual

rend real de # de activo + Prod o Ingr de Act + Transf Gob − Consumo − Tasa Nom el Dinero

Integrando (Cond. de Transv.)

$$\lim_{t \to \infty} a_t e^{-rt} = 0$$

$$(10) \quad a_0 + \int_0^\infty (y_t + z_t) e^{-rt} dt = \int_0^\infty (c_t + i_t m_t) e^{-rt} dt$$

a_0: dot inicial de activos

2.1.2 UTILITY MAXIMIZATION

Household

$$\max_{\{c_t,\, m_t\,,\, \lambda\}_{t=0}^{\infty}} \mathcal{L} = \int_0^{\infty} [\mathrm{u}(c_t) + v(m_t)]\; e^{-\beta t}\; dt \; +$$

$$\lambda \left[a_0 + \int_0^{\infty} (y_t \; + \tau_t)\; e^{-rt}\, dt \; - \int_0^{\infty} (c_t \; + i_t m_t)\, e^{-rt}\, dt \right]$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= u'(c_t)e^{-\beta t} - \lambda\, e^{-rt} = 0 \\ \frac{\partial \mathcal{L}}{\partial m_t} &= v'(m_t)e^{-\beta t} - \lambda\, i_t\, e^{-rt} = 0 \end{aligned}\right\} u'(c_t) = \lambda\, e^{-(r-\beta)t} = \frac{v'(m_t)}{i_t}$$

$$v'(m_t) = u'(c_t) i_t$$

$$m_t = \boldsymbol{L}(c_t\,, i_t)$$

$$+ \quad -$$

Ya que $v'(m_t) = u'(c_t)\, i_t$ (13')

$m_t = L(c_t, i_t)$ (14)

Derivamos (13') respecto a c_t

$$v''(m_t)\cdot \frac{\partial m_t}{\partial c_t} = u''(c_t)\, i$$

$$\frac{\partial m}{\partial c_t} = \frac{\partial L}{\partial c_t} = \frac{u''(c_t)\, i_t}{v''(m_t)} > 0 \quad (15)$$

Derivamos respecto a i_t

$$v''(m_t)\cdot \frac{\partial m_t}{\partial i_t} = u'(c_t)$$

$$\frac{d m}{d i_t} = \frac{\partial L}{\partial i_t} = \frac{u'(c_t)}{v''(m_t)} < 0 \quad (16)$$

- Real money demand is thus increasing in consumption and

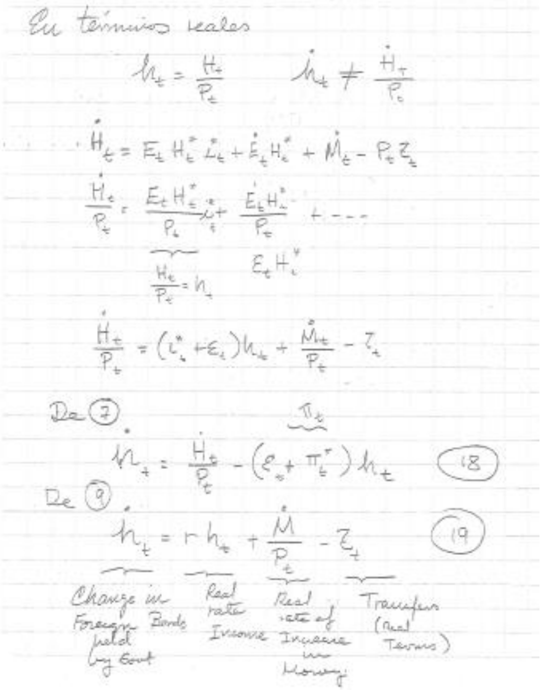
decreasing in the nominal interest rate (the opportunity cost of holding money).

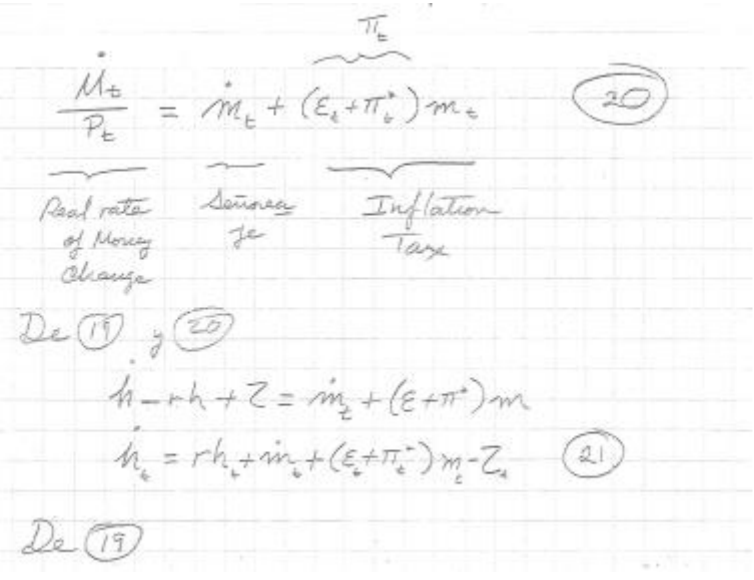
2.2 GOVERNMENT

Define H_t^* the foreign currency value of foreign bonds held by the government (central bank). In domestic terms we have: $H_t = E_t H_t^*$

The government´s flow constraint in nominal terms is given by:

$$\dot{H}_t = \underbrace{i_t^* \, E_t H_t^*}_{\text{Interest Income}} + \underbrace{\dot{E}_t H_t^*}_{\text{Capital gains}} + \underbrace{\dot{M}_t}_{\text{Money printing}} - \underbrace{P_t \tau_t}_{\text{Transfer}} \qquad (17)$$





$$\dot{h}_t = r h_t + \frac{\dot{M}_t}{P_t} - \tau_t \qquad (19)$$

$$h_0 + \int_0^{\infty} \frac{\dot{M}_t}{P_t} e^{-rt}\, dt = \int_0^{\infty} \tau_t e^{-rt}\, dt \qquad (22)$$

2.3 EQUILIBRIUM CONDITIONS

- The assumption of perfect capital mobility implies that the interest parity condition holds:

$$i_t = i_t^* + \varepsilon_t \qquad (23)$$

Let $k_t (\equiv b_t + h_t)$ denote the economy´s stock of net foreign assets.

Combining the consumer´s flow constraint $\dot{a}_t = (i_t^* - \pi_t^*)a_t + y_t + \tau_t - c_t - (i_t^* + \varepsilon_t)\, m_t$ (9) with the government´s $\dot{h}_t = rh_t + \dot{m}_t + (\varepsilon_t + \pi_t^*)m_t - \tau_t$ (21) yields the economy´s flow constraint:

$$\begin{array}{rcl} \dot{a}_t & = & (i_t^* - \pi_t^*)\, a_t + y_t + \tau_t - c_t - (i_t^* + \varepsilon_t)m_t \\ + \\ \dot{h}_t - \dot{m}_t & = & rh_t + (\varepsilon_t + \pi_t^*)m_t - \tau_t \\ \hline \dot{a}_t - \dot{m}_t + \dot{h}_t & = & (i_t^* - \pi_t^*)(a_t - m_t) + rh_t + y_t - c_t \end{array} \qquad (24)$$

$$\underbrace{\dot{b}_t + \dot{h}_t}_{\dot{k}_t} = \underbrace{r_t\, b_t + rh_t}_{rk_t} + \underbrace{y_t - c_t}_{y_t - c_t}$$

The economy accumulates net foreign assets (i.e, $\dot{k_t} > 0$) if the economy´s income $(rk_t + y_t)$ exceeds consumption (c_t). To link the economy´s flows constraint to standard balance of payments accounting, rewrite this equation as:

$$\underbrace{\dot{h}_t}_{\Delta h} = \underbrace{-\dot{b}_t}_{KA} + \underbrace{\underbrace{r(b_t + h_t)}_{IB} + \underbrace{(y_t - c_t)}_{TB}}_{CA}$$

Increase in international reserves = capital account + current account

In Chapter 1 a current account deficit was associated with a capital account surplus, but in a monetary economy can run a current account deficit and a capital account deficit provided that the Central Bank is financing these two deficit by losing international reserves.

Finally, notice that in integrating forward the economy´s flows constraint (equation (24)) and imposing the transversally condition $\lim_{t\to\infty} k_t\, e^{-rt} = 0$ yields the economy´s resource constraint:

$$k_0 + \int_0^\infty y_t\, e^{-rt} dt = \int_0^\infty c_t\, e^{-rt} dt \qquad (26)$$

2.4 PERFECT FORESIGHT EQUILIBRIUM

From $u´(c_t) = \lambda$ along a perfect foresight equilibrium will be constant. Then, from the economy´s resource constraint (26)

$$c = r\,(k_0 + \int_0^{\infty} y_t\, e^{-rt} dt) \qquad (27)$$

Due to the separability of consumption and real money balance in the utility function, consumption is constant over time (and equal to permanent income) regardless of the path of the nominal variable.

- The dynamics of real variables and the same as in Chapter 1.

Hence, in this basic monetary model, money is just a "veil" in the sense that the model exhibits the "dichotomy" between the real and monetary sectors emphasized in classical monetary economics (the monetary sector does not influence the real sector, but real shocks may affect monetary variables).

From $v'(m_t) = u(c_t)i_t \Rightarrow m_t = L\,(c_t, i_t)$ (14), real money demand along a PEEP will be given by

$$m_t = L\,(c, i_t) \qquad (28)$$

Hence real money demand will be constant if the nominal interest rate is constant over time. We now turn to the determination of the nominal interest rate and other nominal variables. In what follows we will assume that π^* is constant over time.

2.5 NOMINAL ANCHORS

Determination of nominal interest rate and the paths of both the nominal exchange rate and the money supply will depend on the specific monetary regime adopted by the monetary authority.

Nominal anchors
- The exchange rate (predetermined exchange rates)
- The money supply (flexible exchange rate)

Consider the Central Bank´s balance sheet:

Assets	**Liabilities**
$\mathrm{E}_t\mathrm{H}_t^*$	M_t
D_t	

To organize thoughts regarding the mechanics of nominal anchors and the determination of nominal magnitudes, it is helpful to rewrite equation (28) as $\frac{M_t}{E_t P_t^*} = L\left(c, i_t\right)$ (30)

Where P_t^* is exogenously-given. In a stationary equilibrium the nominal interest rate, i_t, will be determined by the rate of growth of the nominal anchor.

NOMINAL ANCHORS (CONT.)

<table>
<tr><th></th><th colspan="2">Predetermined</th><th colspan="2">Flexible</th></tr>
<tr><td>E: Exchange rate</td><td>Set by policy</td><td rowspan="5">We can view equations $E_t H_t^* + D_t = M_t$ (29) and $\frac{M_t}{E_t P_t^*} = L\ (c, i_t)$ (30) as a two-equation system with four unknowns: E_t, H_t^*, D_t, and M_t.

Monetary authority sets E_t, then M_t is determined through:
$$\frac{M_t}{E_t P_t^*} = L\ (c, i_t) \qquad (30)$$
With E_t and M_t so determined and D_t being set by the monetary authority, the Central Bank´s balance sheet $E_t H_t^* + D_t = M_t$ (29) endogenously determines H_t^*.</td><td>Endogenous</td><td rowspan="5">Denote with Z the non-monetary liability and rewrite the balance sheet of the Central Bank as: $E_t, H_t^* + D_t = M_t + Z_t$ (31)

We can then view equations (30) and (31) as a system of two equations, in five unknowns: E_t, H_t^* D_t M_t and Z_t.

The Central Bank sets M_t and the money market equilibrium $\frac{M_t}{E_t P_t^*} = L\ (c, i_t)$ sets E_t.

With M_t and E_t so determined the Central Bank sets H_t^* and D_t and equation (31) determines Z_t.</td></tr>
<tr><td>H^*: net foreign bonds</td><td>Endogenous</td><td>Set by policy</td></tr>
<tr><td>D: stock of nominal domestic credit</td><td>Set by policy</td><td>Set by policy</td></tr>
<tr><td>M: money</td><td>Endogenous</td><td>Set by policy</td></tr>
<tr><td>Z: non-monetary liabilities</td><td>N/A</td><td>Endogenous</td></tr>
</table>

PREDETERMINED EXCHANGE RATES

- Setting the path of the nominal exchange rate implies setting the initial level, E_0 and a constant rate of growth, ε , of the nominal exchange rate. Given ε, the interest parity condition $i_t = i_t^* + \varepsilon_t$ (23) determines a constant level of the nominal interest rate, i:

$$i = i^* + \varepsilon \qquad (32)$$

- The constancy of the nominal interest rate implies, from (28) that the real money balance will be constant over time at a level given by

$$m = L\,(c, i) \qquad (33)$$

- Hence, $\dot{m}_t = 0$ for all $t \in [0, \infty)$. Since, by definition, $m_t = \frac{M_t}{E_t P_t^*}$, it follows that the rate of money growth, μ, will also be constant over time:

$$\mu = \varepsilon + \pi^* \qquad (34)$$

- The only nominal variable yet to be determined is the initial level of the nominal money supply, M_0. Since equation (28) holds at time 0, we can write

$$\frac{M_0}{P_0^* E_0} = L\,(c, i)$$

We then solve for M_0:

$$M_0 = P_0^* E_0 L\,(c, i) \qquad (35)$$

PREDETERMINED EXCHANGE RATES (CONT.)

- The monetary authority sets initial level, D_0 , and the (constant) rate of growth (θ) of domestic credit. To determine the resulting path of international reserves, first express the Central Bank's balance sheet in real terms:

$$h_t + d_t = m_t$$

- Then differentiate this identity and solve for $\dot{h}_t$:

$$\dot{h}_t = \dot{m}_t - \dot{d}_t \qquad (36)$$

- Therefore, under predetermined exchange rates, there will be a loss of international reserves, $\dot{h}_t < 0$, when the growth rate of real domestic credit exceeds changes in real money demand, $\dot{d}_t > \dot{m}_t$.
- To gain additional insights, notice that since $d_t \equiv \frac{D_t}{E_t P_t^*}$, $\dot{d}_t = d_t(\theta - \varepsilon - \pi^*)$ (37). Substituting (37) into (36), we obtain

$$\dot{h}_t = \dot{m}_t - d_t(\theta - \varepsilon - \pi^*)$$

- In this particular case, $\dot{m}_t = 0$ because from (33), real money demand is constant. Hence, we can rewrite this equation as:

$$\dot{h}_t = -d_t(\theta - \varepsilon - \pi^*) \qquad (38)$$

- Given that real money demand is constant, if nominal domestic credit is growing faster than domestic inflation (i.e., if $\theta > \varepsilon + \pi^*$), then the monetary authority will be losing international reserves (i.e., $\dot{h}_t < 0$), which has been a common situation in many countries where the monetary authority has been coerced into printing money to finance the fiscal authority's spending. If this is a persistent situation and there is a threshold below which international reserves cannot fall, the monetary authority will run out of international reserves and a balance-of-payments crisis will ensue. Conversely, a situation in which $\theta < \varepsilon + \pi^*$ would imply that the monetary authority accumulates international reserves without bound and so can be ruled out. Hence, for a predetermined exchange rate to be sustainable over time, the rate of domestic credit growth must equal the domestic inflation rate (i.e., if $\theta = \varepsilon + \pi^*$). From (37) this implies that $\dot{d}_t = 0$, and hence from (36),

$$\dot{h}_t = \dot{m}_t - \dot{d}_t \qquad (39)$$

- In equilibrium, changes in real money demand will be reflected in changes in international reserves.

PREDETERMINED EXCHANGE RATES (CONT.)

- Having established that the path of international reserves is constant over time, we need to establish their initial value, h_0. From the Central Bank's balance sheet $(h_t + d_t = m_t)$ at $t = 0$ and $m = L\,(c, i)$ (33), it follows that

$$h_0 = L(c, i) - d_0 \qquad (40)$$

- In principle, h_0, could take any sign, since the Central Bank could have a negative asset position.
- Finally, notice that the variable that adjusts to make the government constraint hold at any point in time is the level of transfers. From $\dot{h}_t = rh_t + \dot{m}_t + (\varepsilon_t + \pi_t^*)m_t - \tau_t$ (21), and taking into account that $h_t, \varepsilon_t,$ and m_t are all constant over time, it follows that

$$\tau = rh + (\varepsilon + \pi^*)m \qquad (41)$$

- To fix ideas, consider a fixed exchange regime $(\varepsilon = 0)$ and zero foreign inflation. Then $\tau = rh$. If $h < 0$, then the government is financing the debt service by taxing the private sector.

FLEXIBLE EXCHANGE RATES

- Under flexible exchange rates the monetary authority does not intervene in the foreign exchange market and allows the exchange rate to be determined by market forces. If the Central Bank does not intervene in the foreign exchange market, its international reserves will be constant over time. For simplicity, it is typically assumed that this initial level of international reserves is zero.
- If international reserves are zero, then the Central Bank's balance sheet reduces to

$$D_t = M_t$$

- Hence, under flexible exchange rates, setting the path of domestic credit is equivalent to setting the path of the money supply. In particular, the monetary authority sets the initial level, M_0, and a constant rate of growth, μ, of the money supply.

FLEXIBLE EXCHANGE RATES (CONT.)

- We first show that real money balances will be constant over time. To see this, notice that $\frac{\dot{m}_t}{m_t} = \mu - \varepsilon_t - \pi^*$, and use $v'(m_t) = \lambda i_t$ (13) and $i_t = i_t^* + \varepsilon_t$ (23) to obtain

$$\dot{m}_t = m_t\left[r + \mu - \frac{v'(m_t)}{\lambda}\right] \qquad (42)$$

- This is a differential equation in m_t because λ is simply some number that has been determined by $u'(c_t) = \lambda$ (12) and $c = r\,(k_0 + \int_0^\infty y_t\, e^{-rt} dt)$ (27). Linearizing this equation around the stationary value for real money balances, denoted by $\bar{m}$ and implicitly given by $r + \mu = \frac{v'(\bar{m})}{\lambda}$, we see that this is an unstable differential equation. Formally,

$$\left.\frac{\partial \dot{m}_t}{\partial m_t}\right|_{\bar{m}} = -\frac{\bar{m}v''(\bar{m})}{\lambda} > 0$$

- This implies that unless m_t is already at its stationary value at $t = 0$, it will diverge over time. Hence, the only convergent equilibrium path is $m_t = \bar{m}$ for all $t \geq 0$. Intuitively, if m_t increases, the nominal interest rate must fall to accommodate this increase. By the interest parity condition, this implies a fall in the rate of depreciation (inflation); this in turn implies that real money supply will grow faster, which requires a further fall in the nominal interest rate, and so forth.

FLEXIBLE EXCHANGE RATES (CONT.)

- Further, since $\frac{\dot{m}_t}{m_t} = \mu - \varepsilon_t - \pi^* = 0$, the (constant) rate of depreciation will be given by

$$\varepsilon = \mu - \pi^* \quad (43)$$

- From the interest parity condition (23) and (43) – and taking into account that $i^* = r + \pi^*$ – it follows that the constant level of the nominal interest rate is given by

$$i = r + \mu \quad (44)$$

- The only nominal variable yet to be determined is the initial level of the nominal exchange rate E_0 (i.e., the initial price level). Since equation (28) holds at time 0, we can write

$$\frac{M_0}{P_0^* E_0} = L\,(c, i)$$

- We then solve for E_0 to obtain

$$E_0 = \frac{M_0}{P_0^* L(c, i)}$$

- We have thus shown that as under predetermined exchange rates, the value if all nominal variables is perfectly well defined under flexible exchange rates.
- Finally, how does the level of transfers get determined? Since international reserves are equal to zero and $\dot{m}_t = 0$, then from (21), the constant level of transfers is given by

$$\tau = (\varepsilon + \pi^*)m \quad (45)$$

3. NEUTRALITY AND SUPERNEUTRALITY

- By "**neutral**" monetary policy, we mean that an unanticipated and permanent change in the level of the money supply has no real effects. It simply leads to an equi-proportional change in the exchange rate.
- By "**neutral**" exchange rate policy means that a permanent devaluation has no real effects and leads to an equi-proportional change in the money supply.
- By "**superneutral**" monetary or exchange rate policy, we mean that neither changes in the rate of money growth nor in the rate of devaluation have any real effects.
- Since both monetary and exchange rate policy are neutral and superneutral, changes in monetary/exchange rate policy will have no effects on the real economy.
- Conversely, real shocks will have the same real effects under either flexible or predetermined exchange rate regimes.
- Hence money is a veil in the basic model in the sense that the economy's real variables are independent of the path of monetary variables.

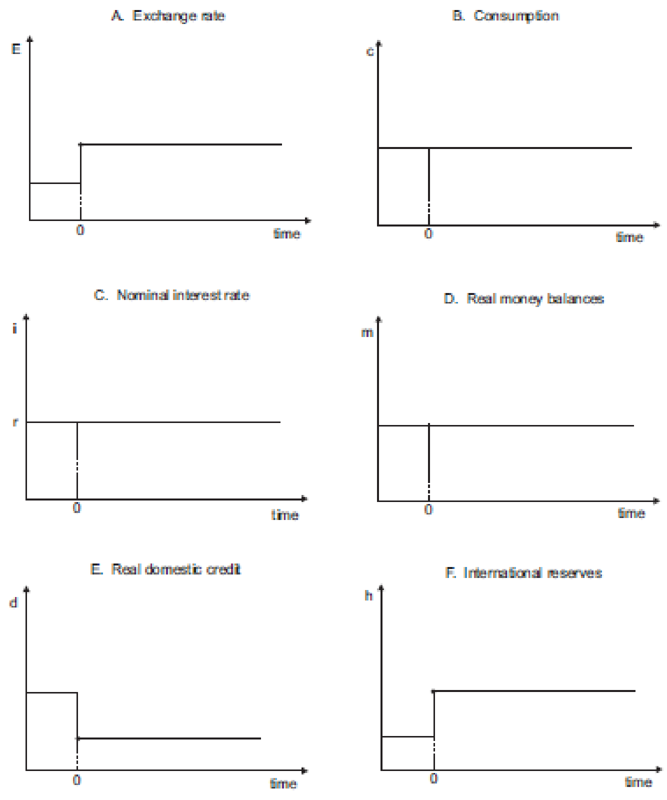
3.1 EXCHANGE RATE POLICY
A PERMANENT DEVALUATION

- Suppose that the economy is initially in the stationary perfect foresight equilibrium characterized above (for $\pi^* = 0$). For simplicity, assume that initially $\varepsilon = \theta = 0$, so that the exchange rate is initially fixed.
- At $t = 0$, there is an unanticipated and permanent increase in the level of the nominal exchange rate (i.e., a permanent devaluation); see Figure 1, Panel A. What will be the effects of this devaluation?
- Clearly, the devaluation has no real effects since we have already shown that, along a PFEP, consumption is given by $c = r\,(k_0 + \int_0^{\infty} y_t\, e^{-rt} dt)$ (27) regardless of the path of the exchange rate (Figure 1, Panel B). From $i = i^* + \varepsilon$ (32), we also see that the nominal interest rate will not change (Figure 1, Panel C). Hence, from $m = L\,(c, i)$ (33), real money demand does not change either (Figure 1, Panel D). From $\mu = \varepsilon + \pi^*$ (34), the same is true of the rate of money growth. From $M_0 = P_0^* E_0 L\,(c, i)$ (35), we see that the initial level of nominal money balances will increase in the same proportion as the exchange rate.
- The main action resulting from a permanent devaluation actually takes place in the Central Bank's balance sheet. Since the stock of nominal domestic credit is controlled by policymakers and hence is given at $t = 0$, the real stock of domestic credit falls at $t = 0$ and remains at that lower level thereafter (Figure 1, Panel E). We can then infer the path of international reserves from the Central Bank balance sheet $(h_t = m_t - d_t)$. It follows that international reserves jump up on impact and remain constant thereafter (Figure 1, Panel F). In fact – as follows from the Central Bank's balance sheet – the change in international reserves at $t = 0$ is exactly equal to the reduction in the real stock of domestic credit:

$$\triangle h_0 = -\triangle d_0 > 0$$

- We thus conclude that *a devaluation leads to a gain in international reserves*.
- What is the intuition behind the increase in international reserves at the Central Bank? The key is that while the devaluation does not affect real money demand, it reduces real money supply for the initial nominal money supply. In other words, at the initial money supply, there is an incipient excess demand for money. To rebuild their money balances, consumers go to the Central Bank and exchange net foreign assets for nominal money balances.

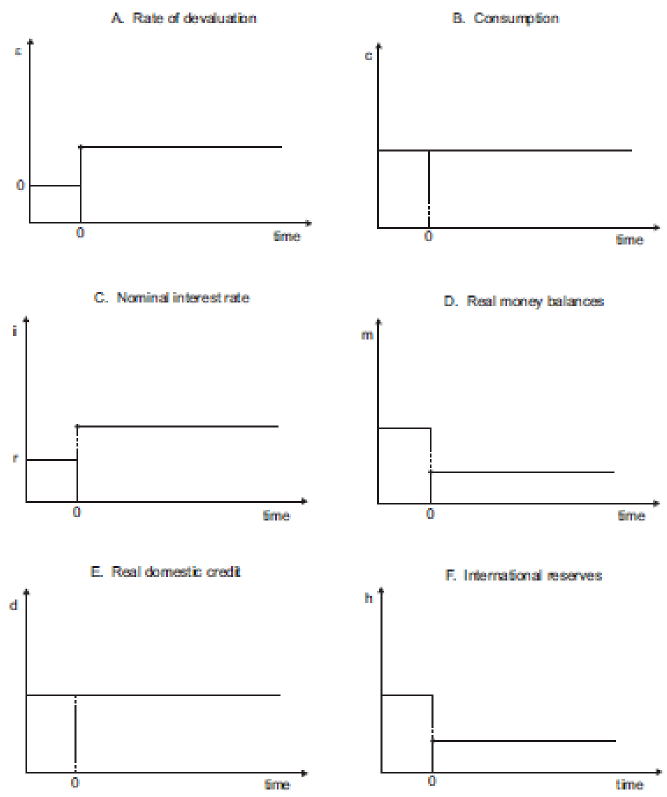
Figure 1. Permanent devaluation



3.1 EXCHANGE RATE POLICY (CONT.)
A PERMANENT INCREASE IN THE RATE OF DEVALUATION

- Suppose now that, starting from the same initial equilibrium (for $\pi^* = \varepsilon = \theta = 0$), there is an unanticipated and permanent increase in the devaluation rate (Figure 2, Panel A). Once again, consumption remains constant (Figure 2, Panel B). From $i = i^* + \varepsilon$ (32), we infer that the nominal interest rate increases pari passu with the devaluation rate (Figure 2, Panel C). Since the opportunity cost of holding money increases, real money demand falls at $t = 0$, as follows from $m = L\,(c, i)$ (33) (Figure 2, Panel D). The rate of money growth increases (from $\mu = \varepsilon + \pi^*$ (34)). From $M_0 = P_0^* E_0 L\,(c, i)$ (35), the initial level of the money supply also falls. The path of real domestic credit remains unchanged (Figure 2, Panel E). Finally, since real money demand falls, we infer from $h_0 = L(c, i) - d_0$ (40) that international reserves fall at $t = 0$ (Panel F).
- How does this adjustment take place? In response to the increase in the opportunity cost of holding money, the consumer wants to reduce his/her real money holdings. To do so, he/she goes to the Central Bank and exchanges domestic money for foreign assets (i.e., sells domestic currency and buys foreign assets). As a result, international reserves at the Central Bank fall whereas private holdings of net foreign assets go up. For the economy as a whole, net foreign assets do not change.
- Note that while a permanent devaluation leads to an increase in international reserves, a permanent increase in the devaluation rate results in a fall in international reserves. These contrasting outcomes are due to the different way in which equilibrium in the money market is affected. In the first case – and for the initial nominal money supply – the devaluation leads to an incipient excess real money demand which requires an upward adjustment in the nominal money supply. In the second case – and for the initial nominal money supply – the increase in the rate of devaluation leads to an incipient excess real money supply which requires a fall in the nominal money supply.

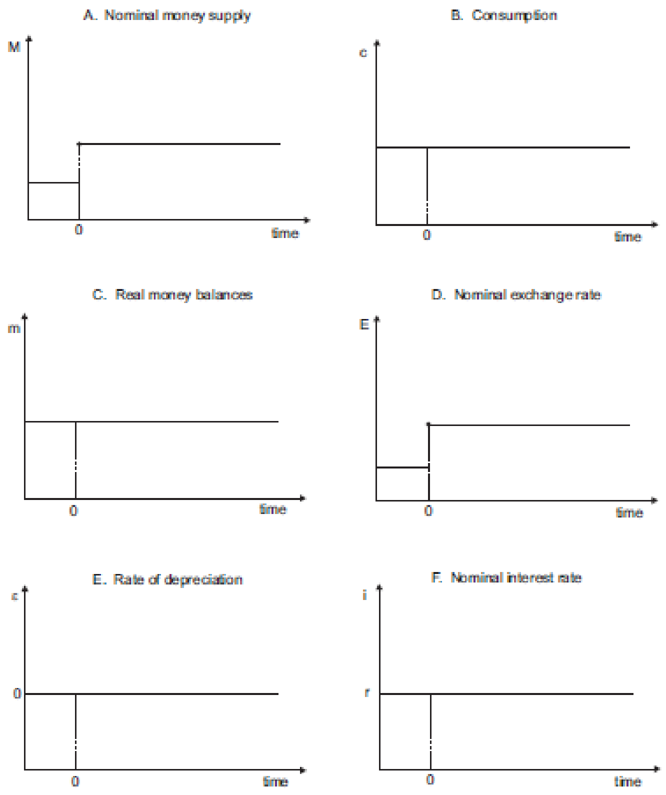
Figure 2. Permanent increase in devaluation rate



3.2 MONETARY POLICY
PERMANENT INCREASE IN THE MONEY SUPPLY

- Suppose that, starting from the initial equilibrium described above (with $\mu = \pi^* = 0$), there is an unanticipated and permanent increase in the nominal money supply (Figure 3, Panel A).
- Consumption, of course, remains constant (Figure 3, Panel B). In the new equilibrium, real money balances must remain constant; otherwise the path of real money balances would diverge over time (Figure 3, Panel C).
- The constancy of real money balances implies that, on impact, the nominal exchange rate will increase by the same proportion as the nominal money supply and remain at that level thereafter (Figure 3, Panel D).
- The fact that $\dot{m}_t = 0$ for all t implies that the rate of depreciation continues to be equal to 0 (Figure 3, Panel E).
- This implies that the nominal interest rate also remains constant (Figure 3, Panel F).
- We conclude that a permanent increase in the nominal money supply leads to an equiproportional increase in the nominal exchange rate, leaving unchanged all other variables.

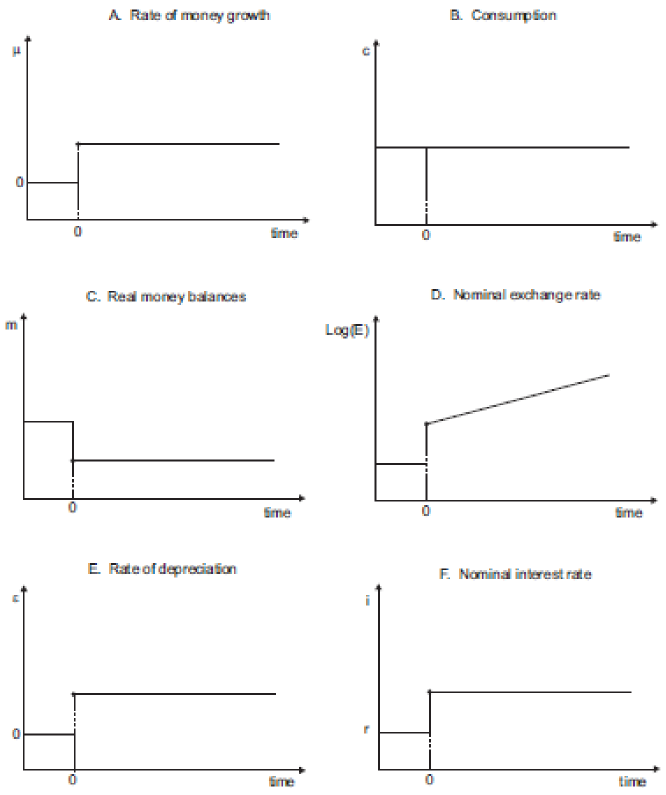
Figure 3. Permanent increase in nominal money supply



3.2 MONETARY POLICY (CONT.)
PERMANENT INCREASE IN THE RATE OF MONEY GROWTH

- Suppose that, starting from the initial equilibrium described above (with $\mu = \pi^* = 0$),, there is an unanticipated and permanent increase in the rate of money growth (Figure 4, Panel A).
- Consumption, of course, remains unchanged (Figure 4, Panel B).
- Since real money balances are governed by an unstable differential equation, they must adjust immediately to their new and lower stationary value (Figure 4, Panel C). Otherwise, the path of real money balances would diverge over time.
- Since the nominal money supply is given at $t = 0$, we infer that the nominal exchange rate increases on impact (Figure 4, Panel D).
- The fact that $\dot{m}_t = 0$ for all t implies that the rate of depreciation increases on impact and remains at that level thereafter (Panel E).
- This, in turn, implies that the nominal exchange rate jumps up at $t = 0$ and then increases at the rate of money growth from $t = 0$ onwards (Figure 4, Panel D).
- By interest parity, the nominal interest rate increases on impact and stays constant thereafter (Figure 4, Panel F).
- We conclude that a permanent increase in the rate of money growth leads to an increase in the exchange rate (a nominal depreciation) and a corresponding increase in the rate of devaluation and the nominal interest rate.

Figure 4. Permanent increase in rate of money growth



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$\frac{\partial \dot{m}_t}{\partial m_t} = \mu^H - \varepsilon_t > 0 \qquad t \in [0, \mathrm{T})$

$\int_0^\infty [u(c_t) + \gamma_t v(m_t)] e^{-\beta t} dt$ (5.46)

$\gamma_t v'(m_t) = \lambda i_t$ (5.47)

$m_t = L(c_t, i_t, \gamma_t)$

$\frac{\partial L}{\partial c_t} = \frac{i_t u''(c_t)}{v''(m_t)\gamma_t} > 0$ (5.48)

$\frac{\partial L}{\partial i_t} = \frac{u'(c_t)}{\gamma_t v''(m_t)} < 0$

$\frac{\partial L}{\partial \gamma_t} = \frac{v'(m_t)}{\gamma_t v''(m_t)} > 0$

$\gamma_t = \begin{cases} \gamma^H, & 0 \leq t < T, \\ \gamma^L, & t \geq T \end{cases} \qquad \gamma^H > \gamma^L$ (5.49)

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$$\dot{m}_t = m_t \left[r + \mu - \frac{\gamma_t v'(m_t)}{\lambda} \right]$$

$$r + \mu \ = m_t \left[\frac{\gamma_t v'(\bar{m}_t)}{\lambda} \right] \qquad (5.50)$$

$$\varepsilon_t = \mu - \ddot{\pi}^* - \frac{\dot{m}_t}{m_t} \qquad (5.51)$$

$$\varepsilon_t = -\frac{1}{m_t^2} (\ddot{m}_t m_t - \dot{m}_t^2 > 0$$

$$M_t^p + E_t B_t^* = E_t(1 + i_{t-1}^*) B_{t-1}^* + M_{t-1} + P_t \tau_t \qquad (5.52)$$

$$M_t^p \geq P_t c_t \qquad (5.53)$$

$$M_t = M_t^p - P_t c_t + P_t y_t \qquad (5.54)$$

$$M_t + E_t B_t^* = E_t(1 + i_{t-1}^*)B_{t-1}^* + M_{t-1} + P_t \tau_t + P_t\, y_t - P_t c_t \qquad (5.55)$$

$$A_t + A_{t-1} = \underbrace{E_t i_{t-1}^* B_{t-1}^*}_{Interest\ Income} + \underbrace{(E_t - E_{t-1})B_{t-1}^*}_{Capital\ gains} + P_t\, \tau_t + P_t y_t - P_t c_t$$

$$m_t + b_t = \frac{E_t}{P_t} P_{t-1}^*(1 + i_{t-1}^*)b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + \tau_t + y_t - c_t$$

$$m_t + b_t = (1 + r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t + y_t - c_t \qquad (5.56)$$

$$a_t = (1 + r)a_{t-1} + \tau_t + y_t - c_t - \frac{i_{t-1}}{1+\pi_t} m_{t-1} \qquad (5.57)$$

$$E_t(1 + i_{t-1}^*)B_{t-1}^* + M_{t-1} + P_t \tau_t - E_t B_t^* \geq P_t c_t \qquad (5.58)$$

$$(1+r)b_{t-1}+\frac{m_{t-1}}{1+\pi_t}+\tau_t-b_t\geq c_t \qquad (5.59)$$

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^t u(c_t)$$

$$+\sum_{t=0}^{\infty}\beta^t\lambda_t\left[(1+r)b_{t-1}+\frac{m_{t-1}}{1+\pi_t}+\tau_t+y_t-c_t-b_t-m_t\right]$$

$$+\sum_{t=0}^{\infty}\beta^t\psi_t\left[(1+r)b_{t-1}+\frac{m_{t-1}}{1+\pi_t}+\tau_t-b_t-c_t\right]$$

$$u'(c_t)=\lambda_t+\psi_t \qquad (5.60)$$

$$-\lambda_t+\frac{\beta\lambda_{t+1}}{1+\pi_{t+1}}+\frac{\beta\psi_{t+1}}{1+\pi_{t+1}}=0 \qquad (5.61)$$

$$\beta(1+r)(\lambda_{t+1}+\psi_{t+1})=\lambda_t+\psi_t \qquad (5.62)$$

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$$\left[(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t - c_t\right]\Psi = 0 \qquad (5.63)$$

$$u'(c_{t+1}) = u'c_t \qquad (5.64)$$

$$h_t = (1+r)h_{t-1} + \frac{M_t - M_{t-1}}{P_t} - \tau_t \qquad (5.65)$$

$$\frac{M_t - M_{t-1}}{P_t} = m_t - m_{t-1} + \frac{\pi_t}{1+\pi_t} m_{t-1} \qquad (5.66)$$

$$1 + i_t = (1 + i_t^*)\frac{E_t+1}{E_t} \qquad (5.67)$$

$$b_t + h_t = (1+r)(b_{t-1} + h_{t-1}) + y_t - c_t \qquad (5.68)$$

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = (1+r)(b_{-1} + h_{-1}) + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \qquad (5.69)$$

$$c = \frac{r}{1+r}\left[(1+r)(b_{-1} + h_{-1}) + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}\right] \qquad (5.70)$$

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$$\frac{M_t^p}{P_t} = c \qquad (5.71)$$

$$M_t = P_t y_t \qquad (5.72)$$

$$1 + i_t = (1 + i_t^*)(1 + \varepsilon) \qquad (5.73)$$

$$\frac{P_t+1}{P_t} = \frac{1+\mu}{y_{t+1}/y_t} \qquad t \geq 0 \qquad (5.74)$$

$$E_t = \frac{P_t}{P_t^*} \qquad (5.75)$$

$$\frac{E_{t+1}}{E_t} = \frac{P_{t+1}/P_t}{1+\pi^*} \qquad (5.76)$$

$$1 + i_t = (1 + r)\left(\frac{1+\mu}{y_{t+1}/y_t}\right) \qquad (5.77)$$

CHAPTER 5: EXERCISES

1. Demand shocks

This exercise shows that, as one would expect, the dichotomy between the real and the monetary sectors is still valid when the path of consumption is not constant over time. To this effect, consider the following variation of the model in the text. Preferences are given by:

$$\int_0^\infty [\alpha_t \mathrm{u}(c_t) + v(m_t)]\ e^{-\beta t}\ dt$$

where α_t is a preference shock. The rest of the model is unchanged. The parameter α_t can be viewed as a demand shock. Suppose that the path of α_t is as follows:

$$\alpha_t = \begin{cases} \alpha^H & 0 \leq t < T \\ \alpha^L & t \geq T \end{cases}$$

where $\alpha^H > \alpha^L$. In this context:

(a) Solve for the perfect foresight equilibrium path corresponding to predetermined exchange rates.

(b) Solve for the perfect foresight equilibrium path corresponding to flexible exchange rates and show that it coincides with the one you just derived for predetermined exchange rates.

CHAPTER 5: EXERCISES (CONT.)

2. Increase in domestic credit

Consider the economy analyzed in the text operating under predetermined exchange rates. Analyze the effects of an unanticipated and permanent increase in the stock of domestic credit at time 0, d_0.

CHAPTER 5: EXERCISES (CONT.)

3. Equivalence between predetermined and flexible rates illustrated with an anticipated increase in the level of the money supply

Consider the economy analyzed in the text operating under flexible exchange rates. Suppose that the rate of money growth is zero (i.e., $\mu = 0$) and that the level of the money supply follows the path given by:

$$M_t = \begin{cases} M^L & 0 \leq t < T \\ M^H & t \geq T \end{cases}$$

where $M^L < M^H$.

In this context:

(a) Solve for the perfect foresight path of all relevant variables.

(b) Show that if the economy were operating under predetermined exchange rates and the central bank set the path of the nominal exchange rate.

CHAPTER 5: EXERCISES (CONT.)

4. Inflationary consequences of anticipated changes in policy

This exercise explores yet another important distinction between a predetermined and flexible exchange rate systems: the behavior of the inflation rate in response to an anticipated changes in policy.

Consider the model of Section 2. Characterize the perfect foresight equilibrium paths corresponding to the following cases:

(a) Under predetermined exchange rates, suppose that the rate of devaluation is zero between 0 and T and increases to $\bar{\varepsilon} > 0$ at $t = T$. Solve for the path of all relevant variables.

(b) Under flexible exchange rates, suppose that the rate of money growth is zero between 0 and T and increases to $\bar{\mu} > 0$ at time T. Solve for the path of all relevant variables.

(c) How does the behavior of inflation differ? What is the intuition behind the results?

CHAPTER 5: EXERCISES (CONT.)

5. Dirty floating

This exercise illustrates how one would think about "dirty floating" in the monetary model analyzed in the main section. Specifically, we analyze how the economy would respond to positive monetary shock that would lead to an appreciation of the domestic currency and how the monetary authority (MA) might intervene to partly offset such an appreciation (perhaps because, for reasons left out of the model, the MA fears that a large appreciation might worsen the trade balance).

Consider the model of Section 2 with the only modification that preferences are now given by:

$$\int_0^\infty [\mathrm{u}(c_t) + \alpha_t v(m_t)]\ e^{-\beta t}\ dt$$

where α_t should be thought of as a money demand shock. In the context of this model:

(a) Consider the case of flexible exchange rates (with $\mu = 0$). Suppose that just before $t = 0$, the economy is in a stationary equilibrium with a constant α. At $t = 0$, there is unanticipated and permanent increase in α. Solve for the non-intervention case (i.e., a "pure floating"). Explain the intuition behind the results.

(b) Solve for the extreme case of "full intervention"(i.e., the MA reacts in such a way that it does not let the nominal exchange rate change). Explain intuitively how this policy works.

(c) Consider an "intermediate case" in which the MA chooses to intervene in the foreign exchange market (but allows some of the adjustment to take place through the nominal exchange rate). In particular, derive a "policy reaction function" that would tell the MA how much to intervene as a function of the change in real money demand (which the MA must take as given) and the targeted change in the nominal exchange rate. [HINT: a) Think of small changes so that you can use differentiation to compute changes at $t = 0$. b) Think of the MA as having an initial positive stock of international reserves and that capital gains/losses reserves are not monetized; that is, there is some non-monetary liability (call it NM) that is adjusted.]